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PIEZOELECTRIC–ELECTROMAGNETIC HYBRID ENERGY HARVESTING SYSTEM: WHEN IS IT USEFUL?

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ABSTRACT

We investigate the performance of a hybrid energy harvesting system composed of piezoelectric and electromagnetic transducers. We introduce effective figures of merit for the piezoelectric and electromagnetic generators that combine the transducer coupling and resistive loss. The maximum output power of single–transducer and hybrid systems are determined analytically, expressed as functions of effective figures of merit. A solution of the conditions under which a resonant hybrid harvester system with multiple transduction mechanisms can outperform its counterpart that uses a single energy conversion is derived. We also find that there is no benefit to utilizing a hybrid system if one of the two, or both, effective figures of merit exceeds a threshold of $\mathcal{M}^* \approx 2.38$.

KEYWORDS

Power Optimization, Energy Harvesting, Wireless Power Transfer, Multi–mechanism System.

INTRODUCTION

Energy harvesting technology has exploded in the last two decades as an alternative to conventional power sources for low–power electronics. It is becoming a key enabling technology for various applications, ranging from structural health monitoring to wearable and implantable devices, and for the future internet of things [1]. Until recently, an energy harvester typically utilized a single energy conversion mechanism. However, hybrid energy harvesters are attracting more and more attention [2]. These hybrid systems can be classified into two categories, multi–source energy harvesters and single–source harvesters with multiple mechanisms. In this work, we narrow our focus on the latter.

A hybrid energy harvester is often expected to improve the space utilization efficiency and, at the same time, increase the total output power [3, 4]. However, it is unclear under what circumstances, if any, introducing more than one transduction mechanism is beneficial. Addressing this concern is the central objective of the paper. Furthermore, to reduce the complexity of a hybrid system, none of the previous work has considered the parasitic loss of the piezoelectric and electromagnetic transducers simultaneously. In contrast, we account for both loss factors to better model a realistic hybrid device.

We choose a vibration energy harvesting system that includes piezoelectric and electromagnetic transducers as an example of the study. We consider a common, single mass for the transducers and linear loading. The approach we use here is also applicable to other similar energy harvesting or wireless power transfer systems.

PIEZOELECTRIC–ELECTROMAGNETIC HYBRID SYSTEMS: MATHEMATICAL MODEL

An example structure of a hybrid energy harvesting system composed of piezoelectric and electromagnetic transducers is illustrated in Figure 1. A permanent magnet is mounted at the tip of a cantilever that is a bimorph piezoelectric composite beam. Two piezoelectric layers are poled in opposite directions and connected in series. A pickup coil is placed in proximity to the magnet mass, forming an electromagnetic generator. Under an external excitation, the mass vibrates in the transverse direction, and its kinetic energy can be converted to electricity simultaneously through the piezoelectric effects and Faraday's law of induction. It is important to note that the excitation can be vibration or magnetic fields. In the latter case, the interactions between the magnet and the **B**–field or the magnetic flux gradient create a moment or a force acting on the resonator. Therefore, although we focus on energy harvesting systems, the structure under consideration can represent three different device types, a vibration energy harvester, a magnetic energy harvester, and a receiver for a low–frequency wireless power transfer system [5-7].

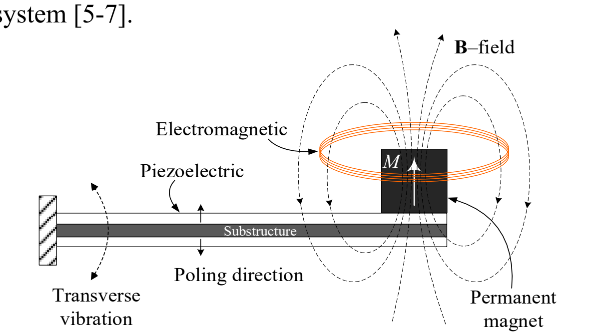

Figure 1: Schematic of a piezoelectric–electromagnetic hybrid system.

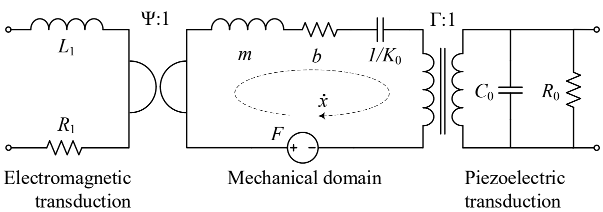

Figure 2: Equivalent circuit of linear three–port model.

The system in Figure 1 can be described by a linear three–port model whose equivalent circuit is shown in Figure 2 [8]. m, b, and K_0 are the effective mass, mechanical damping coefficient, and mechanical stiffness. The equivalent drive force has the form of $F = F_0 \cos(\omega t)$ where ω is the operating angular frequency. Ψ and Γ are the electromagnetic and electromechanical transduction factors. L_1 and C_0 are the inductance of the pickup coil and the equivalent capacitance of the piezoelectric composite. The model also includes inevitable parasitic losses of the two transducers, characterized by two resistances, one in series with the coil inductance and the other in parallel with the piezoelectric capacitance, denoted as R_1 and R_0, respectively. The effects of the parasitic capacitance in parallel with L_1 are negligible in the frequencies of interest.

POWER OPTIMIZATION FOR EACH SINGLE TRANSDUCER

Only one transduction of energy conversion is active when the output terminals of the other one are open (for the electromagnetic transduction) or shorted (for the piezoelectric transduction). The electromagnetic transducer is taken as an example, but the results hold for both types of generators.

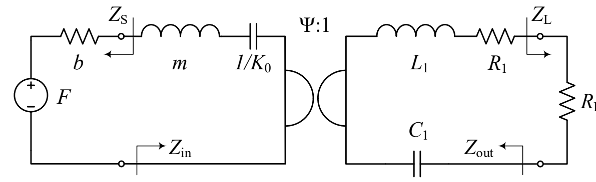

Figure 3: Equivalent circuit for electromagnetic transducer.

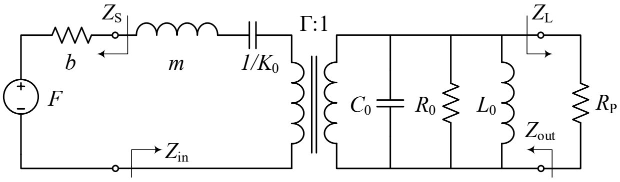

Figure 4: Equivalent circuit for piezoelectric transducer.

The equivalent circuit model for the case where only electromagnetic transduction is active is shown in Figure 3. A load resistance R_{E} is connected to the electrical port for the sake of simplification. The maximum possible output power delivered to the load can be found by applying network theory [9, 10]. We consider the bi–conjugate impedance matching condition

$$Z_{\mathrm{in}} = Z_{\mathrm{S}}^{*}, Z_{\mathrm{out}} = Z_{\mathrm{L}}^{*} \quad (1)$$

where $Z_{\mathrm{in}}, Z_{\mathrm{out}}, Z_{\mathrm{L}}$, and Z_{S} are defined as in Figure 3. In this case, the equivalent impedance relations are

$$\Im\{Z_{\mathrm{in}}\} = \Im\{Z_{\mathrm{out}}\} = 0, \quad (2)$$
$$\Re\{Z_{\mathrm{in}}\} = b, \quad (3)$$
$$\Re\{Z_{\mathrm{out}}\} = R_{\mathrm{E}}. \quad (4)$$

One approach to realize the impedance matching is to connect a capacitor C_1 in series with the coil L_1 to form a resonator. Solving (2) yields the following cases.

Case I where $\omega = \sqrt{K_0/m} = 1/\sqrt{L_1 C_1}$. The solution of (4) is

$$\tau_{\mathrm{E}} = \frac{\tau_1}{1+\mathcal{M}_{\mathrm{E}}} \quad (5)$$

where $\tau_{\mathrm{E}} = L_1/R_{\mathrm{E}}, \tau_1 = L_1/R_1$ and $\mathcal{M}_{\mathrm{E}} = \Psi^2/(bR_1)$. Substituting (5) into (3) leads to $bR_1 = 0$, whose physical solution does not exist. The mismatch between b and $\Re\{Z_{\mathrm{in}}\}$ is given by $b/\Re\{Z_{\mathrm{in}}\} = (\mathcal{M}_{\mathrm{E}} + 2)/\mathcal{M}_{\mathrm{E}} > 1$. The output power in Case I is

$$P_{\mathrm{E}} = \frac{F_0^2}{8b}\frac{\mathcal{M}_{\mathrm{E}}}{\mathcal{M}_{\mathrm{E}}+1}. \quad (6)$$

By definition, the squared electromagnetic coupling coefficient is given by $k_{\mathrm{E}}^2 = \Psi^2/(K_{\mathrm{E}}L_1)$ where $K_{\mathrm{E}} = K_0 + \Delta K_{\mathrm{E}}$ and $\Delta K_{\mathrm{E}} = \Psi^2/L_1$. The expedient coupling coefficient (or the generalized coupling coefficient) is defined as $k_{\mathrm{e,E}}^2 = k_{\mathrm{E}}^2/(1-k_{\mathrm{E}}^2)$. Thus, we arrive at $\Psi^2 = k_{\mathrm{e,E}}^2 K_0 L_1 = k_{\mathrm{e,E}}^2 m\omega_0^2 L_1$. Introducing the mechanical quality factor and the coil quality factor at resonance frequency ω_0, $Q_0 = \omega_0 m/b$ and $Q_{\mathrm{L}} = \omega_0 L_1/R_1$, we can write

$$\mathcal{M}_{\mathrm{E}} = \frac{\Psi^2}{bR_1} = k_{\mathrm{e,E}}^2\left(\frac{m\omega_0}{b}\right)\left(\frac{\omega_0 L_1}{R_1}\right) = k_{\mathrm{e,E}}^2 Q_0 Q_{\mathrm{L}}. \quad (7)$$

The parameter $M = k_{\mathrm{e,E}}^2 Q_0$ is usually referred to as a *resonator figure of merit*, whose more general form is $M = \Delta K_{\mathrm{E}}/(\omega b)$ [11]. Therefore, $\mathcal{M}_{\mathrm{E}}$ can be considered as an *effective figure of merit* of the electromagnetic transducer with the presence of the parasitic resistance R_1.

For Case II, we obtain

$$\omega m - \frac{K_0}{\omega} = \pm\left(\frac{\Psi^2 b}{R_1+R_{\mathrm{E}}} - b^2\right)^{1/2}, \quad (8)$$
$$\omega L_1 - \frac{1}{\omega C_1} = \pm\left[\frac{\Psi^2(R_1+R_{\mathrm{E}})}{b} - (R_1+R_{\mathrm{E}})^2\right]^{1/2}. \quad (9)$$

The right–hand side of this solution is only real for $R_{\mathrm{E}} \leq R_{\mathrm{E}}^{*} = \Psi^2/b - R_1$. The physical solutions of ω and C_1 can be determined as follows

$$\omega = \frac{X+\sqrt{4K_0 m + X^2}}{2m}, \quad (10)$$
$$C_1 = (\omega(\omega L_1 - Y))^{-1} \quad (11)$$

where X and Y are the right–hand sides of (8) and (9), respectively. C_1 is positive only for $\omega L_1 > Y$, which holds for all $Y < 0$. The obtained solution in (8) and (9) also fulfills condition (3). However, equation (4) now gives $R_1 = 0$, which is unphysical. In other words, condition (4) cannot be fulfilled.

In Case II, we note that both ω and C_1 are functions of R_{E}, and for all $R_{\mathrm{E}} \leq R_{\mathrm{E}}^{*}$, there always exists ω and C_1 such that (2) and (3) are satisfied. The corresponding output power in terms of R_{E} is derived as

$$P_{\mathrm{E}} = \frac{F_0^2}{8b}\frac{R_{\mathrm{E}}/R_1}{R_{\mathrm{E}}/R_1+1}. \quad (12)$$

This power expression is only valid for $R_{\mathrm{E}} \leq R_{\mathrm{E}}^{*}$, which is equivalent to $R_{\mathrm{E}}/R_1 \leq \Psi^2/(R_1 b) - 1 = \mathcal{M}_{\mathrm{E}} - 1 < \mathcal{M}_{\mathrm{E}}$. Therefore, the power in (12) is always less than that in (6), due to the increasing property of function $f(x) = x/(x+1)$ with $x > 0$.

For a piezoelectric harvester, its equivalent circuit model is depicted in Figure 4. The previous result obtained for electromagnetic generator is applicable to piezoelectric transducer. In particular,

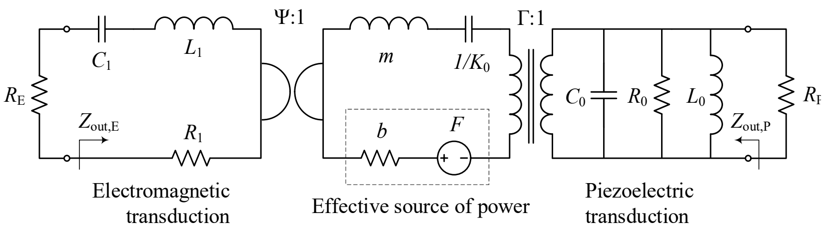

Figure 5: Equivalent circuit model for a piezoelectric–electromagnetic hybrid system.

$$P_{\mathrm{P}} = \frac{F_0^2}{8b}\frac{\mathcal{M}_{\mathrm{P}}}{\mathcal{M}_{\mathrm{P}}+1} \tag{13}$$

where $\mathcal{M}_{\mathrm{P}} = \Gamma^2 R_0/b$. In analogy to $\mathcal{M}_{\mathrm{E}}$, we can also write $\mathcal{M}_{\mathrm{P}} = k_{\mathrm{e,P}}^2 Q_0 Q_{\mathrm{C}}$ where $Q_{\mathrm{C}} = \omega_0 R_0 C_0$. For the piezoelectric generator, the electromechanical coupling coefficient is defined as $k_{\mathrm{P}}^2 = \Gamma^2/(K_{\mathrm{P}} C_0)$ where $K_{\mathrm{P}} = \Delta K_{\mathrm{P}} + K_0$ and $\Delta K_{\mathrm{P}} = \Gamma^2/C_0$. We have that $P_{\mathrm{E}} \geq P_{\mathrm{P}}$ if and only if $\mathcal{M}_{\mathrm{E}} \geq \mathcal{M}_{\mathrm{P}}$, or equivalently, $\Psi/\Gamma \geq \sqrt{R_0 R_1}$.

In brief, the maximum possible power that the electromagnetic and piezoelectric generators can provide are given by (6) and (13), respectively. We find that an alternative approach by means of the network theory and reflected impedance yields the same limit on power.

POWER OPTIMIZATION FOR A HYBRID SYSTEM

We now consider a general case in which both generators operate simultaneously, whose equivalent circuit model is presented Figure 5. The impedance matching theory is utilized to derive the output power of each mechanism, and their summation yields the total maximum power, P_{T}. The input impedance Z_{in} seen from the effective power source and the output impedance $Z_{\mathrm{out,P}}$ and $Z_{\mathrm{out,E}}$ seen by the piezoelectric and electromagnetic generators are given by

$$\begin{aligned}
Z_{\mathrm{in}} &= j\omega m + K_0/(j\omega) \\
&\quad + \Gamma^2 (R_0^{-1} + R_{\mathrm{P}}^{-1} + (j\omega L_0)^{-1} + j\omega C_0)^{-1} \\
&\quad + \Psi^2/(R_1 + R_{\mathrm{E}} + j\omega L_1 + (j\omega C_1)^{-1}), \\
Z_{\mathrm{out,P}}^{-1} &= R_0^{-1} + (j\omega L_0)^{-1} + j\omega C_0 \\
&\quad + \Gamma^2 (b + j\omega m + K_0/(j\omega) \\
&\quad + \Psi^2 (R_{\mathrm{E}} + R_1 + j\omega L_1 + (j\omega C_1)^{-1})^{-1})^{-1}, \\
Z_{\mathrm{out,E}} &= R_1 + j\omega L_1 + (j\omega C_1)^{-1} \\
&\quad + \Psi^2 (b + j\omega m + K_0/(j\omega) \\
&\quad + \Gamma^2 (R_{\mathrm{P}}^{-1} + R_0^{-1} + (j\omega L_0)^{-1} + j\omega C_0)^{-1})^{-1}.
\end{aligned}$$

Following the approach presented in Section 3, the drive frequency ω and the external components L_0 and C_1 are chosen such that $\omega = \sqrt{K_0/m} = 1/\sqrt{L_0 C_0} = 1/\sqrt{L_1 C_1}$. This choice makes $\Im\{Z_{\mathrm{in}}\} = \Im\{Z_{\mathrm{out,E}}\} = \Im\{Z_{\mathrm{out,P}}\} = 0$. The optimal loads are then obtained by matching to the real part of the corresponding output impedances, $R_{\mathrm{E}} = \Re\{Z_{\mathrm{out,E}}\}$ and $R_{\mathrm{P}} = \Re\{Z_{\mathrm{out,P}}\}$. We get

$$\begin{aligned}
\tau_{\mathrm{P}} &= \tau_0 \sqrt{\frac{\mathcal{M}_{\mathrm{E}}+1}{(\mathcal{M}_{\mathrm{P}}+1)(\mathcal{M}_{\mathrm{P}}+\mathcal{M}_{\mathrm{E}}+1)}}, \\
\tau_{\mathrm{E}} &= \tau_1 \sqrt{\frac{\mathcal{M}_{\mathrm{P}}+1}{(\mathcal{M}_{\mathrm{E}}+1)(\mathcal{M}_{\mathrm{E}}+\mathcal{M}_{\mathrm{P}}+1)}}.
\end{aligned}$$

In a special case where

$$\mathcal{M}_{\mathrm{E}} = \frac{(\mathcal{M}_{\mathrm{P}}+1)\sqrt{\mathcal{M}_{\mathrm{P}}^2+4} - (\mathcal{M}_{\mathrm{P}}-1)(\mathcal{M}_{\mathrm{P}}+2)}{2\mathcal{M}_{\mathrm{P}}},$$

the condition (3) is satisfied and (1) is fully fulfilled.

The corresponding output power of each mechanism is given by

$$\begin{aligned}
P_{\mathrm{P}} &= \frac{F_0^2}{8b}\frac{(\sqrt{\mathcal{M}_{\mathrm{P}}+1}\sqrt{\mathcal{M}_{\mathrm{P}}+\mathcal{M}_{\mathrm{E}}+1} - \sqrt{\mathcal{M}_{\mathrm{E}}+1})^2}{\mathcal{M}_{\mathrm{P}}\sqrt{\mathcal{M}_{\mathrm{P}}+1}\sqrt{\mathcal{M}_{\mathrm{E}}+1}\sqrt{\mathcal{M}_{\mathrm{P}}+\mathcal{M}_{\mathrm{E}}+1}}, \\
P_{\mathrm{E}} &= \frac{F_0^2}{8b}\frac{(\sqrt{\mathcal{M}_{\mathrm{E}}+1}\sqrt{\mathcal{M}_{\mathrm{E}}+\mathcal{M}_{\mathrm{P}}+1} - \sqrt{\mathcal{M}_{\mathrm{P}}+1})^2}{\mathcal{M}_{\mathrm{E}}\sqrt{\mathcal{M}_{\mathrm{E}}+1}\sqrt{\mathcal{M}_{\mathrm{P}}+1}\sqrt{\mathcal{M}_{\mathrm{E}}+\mathcal{M}_{\mathrm{P}}+1}},
\end{aligned}$$

which yields the total output power

$$P_{\mathrm{T}} = \frac{F_0^2}{4b}\left(\frac{1}{\mathcal{M}_{\mathrm{E}}} + \frac{1}{\mathcal{M}_{\mathrm{P}}}\right)\left[\frac{\sqrt{\mathcal{M}_{\mathrm{P}}+1}\sqrt{\mathcal{M}_{\mathrm{E}}+1}}{\sqrt{\mathcal{M}_{\mathrm{E}}+\mathcal{M}_{\mathrm{P}}+1}} - 1\right]. \tag{14}$$

It is important to note that (14) reduces to (6) or (13) if only either the electromagnetic or piezoelectric generator is active. In particular, for example,

$$\lim_{\mathcal{M}_{\mathrm{E}}\to 0} P_{\mathrm{T}} = \frac{F_0^2}{8b}\frac{\mathcal{M}_{\mathrm{P}}}{\mathcal{M}_{\mathrm{P}}+1}. \tag{15}$$

This verifies the consistency of (6), (13), and (14).

The method of forming resonators by connecting the capacitor and inductor in series or parallel used in Sections 3 and 4 is a convenient mathematical approach to determine the maximum output power. However, the required capacitance or inductance might be too large to be feasible, especially for low–frequency systems. Therefore, optimizing the load resistance and the operating frequency could be of more interest in practice.

WHEN IS UTILIZING A MULTI–MECHANISM SYSTEM HELPFUL?

The conditions under which a multi–mechanism system is preferable over a single transduction configuration (in terms of power) are solved based on the inequality formed by the power expressions obtained from the two cases. Without loss of generality, we consider the inequality

$$P_{\mathrm{T}} \geq P_{\mathrm{E}} \tag{16}$$

where P_{T} and P_{E} are given in (14) and (6), respectively. With both $\mathcal{M}_{\mathrm{E}}$ and $\mathcal{M}_{\mathrm{P}}$ being positive, (16) reduces to

$$\begin{aligned}
&(-\mathcal{M}_{\mathrm{E}}^3 + 4\mathcal{M}_{\mathrm{E}} + 4)\mathcal{M}_{\mathrm{P}} \\
&-\mathcal{M}_{\mathrm{E}}(\mathcal{M}_{\mathrm{E}}+1)(\mathcal{M}_{\mathrm{E}}-2)(\mathcal{M}_{\mathrm{E}}+2) \geq 0.
\end{aligned} \tag{17}$$

Solutions to the inequality are summarized as follows

$$\begin{aligned}
\mathcal{M}_{\mathrm{P}} \geq \quad & \mathcal{M}_{\mathrm{t}} = \frac{\mathcal{M}_{\mathrm{E}}(\mathcal{M}_{\mathrm{E}}+1)(\mathcal{M}_{\mathrm{E}}-2)(\mathcal{M}_{\mathrm{E}}+2)}{-\mathcal{M}_{\mathrm{E}}^3+4\mathcal{M}_{\mathrm{E}}+4} \\
& \forall \mathcal{M}_{\mathrm{E}} \in [2, \mathcal{M}^*)
\end{aligned} \tag{18}$$

where $\mathcal{M}^* = \frac{1}{3}\left(\sqrt[3]{6(9-\sqrt{33})} + \sqrt[3]{6(9+\sqrt{33})}\right) \approx 2.38,$

and

$$\mathcal{M}_\mathrm{P} > 0 \;\; \forall \mathcal{M}_\mathrm{E} \in (0, 2). \tag{19}$$

Due to the symmetry in the roles of $\mathcal{M}_\mathrm{E}$ and $\mathcal{M}_\mathrm{P}$, the solutions (18) and (19) hold when exchanging $\mathcal{M}_\mathrm{P}$ for $\mathcal{M}_\mathrm{E}$. If any of $\mathcal{M}_\mathrm{P}$ or $\mathcal{M}_\mathrm{E}$ exceeds $\mathcal{M}^*$, P_T is always less than P_P or P_E, and therefore, a hybrid system is not beneficial. On the contrary, a hybrid is helpful when $\mathcal{M}_\mathrm{i}$ is equal or higher than $\mathcal{M}_\mathrm{t}$ for all $2 \leq \mathcal{M}_\mathrm{j} < \mathcal{M}^*$ and any $\mathcal{M}_\mathrm{i} > 0$ for all $0 < \mathcal{M}_\mathrm{j} < 2$, where $(\mathrm{i}, \mathrm{j}) = \{(\mathrm{P}, \mathrm{E}) \vee (\mathrm{E}, \mathrm{P})\}$.

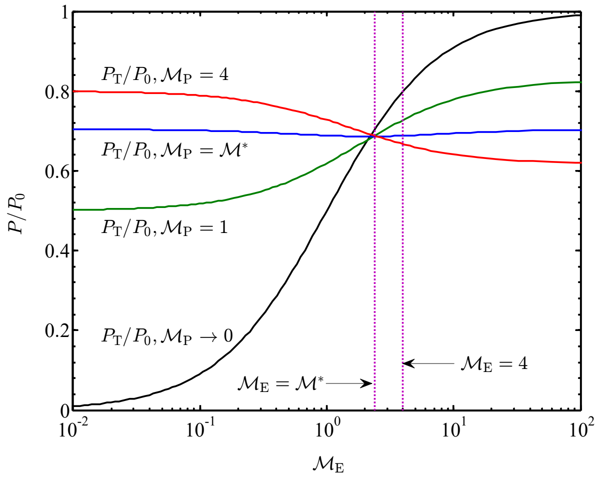

Figure 6: Comparison between the output power of single– and multiple–transduction systems.

A visualization of the findings is presented in Figure 6. We note that $P_\mathrm{P} = P_\mathrm{E}$ when $\mathcal{M}_\mathrm{P} = \mathcal{M}_\mathrm{E}$, and Figure 6 keeps unchanged if the two quantities $\mathcal{M}_\mathrm{P}$ and $\mathcal{M}_\mathrm{E}$ are swapped. Let us consider the case when the effective figure of merit of the piezoelectric transducer is fixed at $\mathcal{M}_\mathrm{P} = 4 > \mathcal{M}^*$ as an example (i.e., red solid line in the figure). If the harvester only contains a single piezoelectric transducer, we have that $P_\mathrm{P}/P_0 = 4/5 = 0.8$ due to (13). Here, we define $P_0 = F_0^2/(8b)$. The maximum output power of a hybrid system P_T, given by (14), is always less than P_P. $P_\mathrm{T} \to P_\mathrm{P}$ only if $\mathcal{M}_\mathrm{E} \to 0$, and P_T decreases with the increase of $\mathcal{M}_\mathrm{E}$. Therefore, a hybrid system is not preferable to a single transduction device under this circumstance.

In general, if a harvester cannot provide sufficiently strong coupling with a single transduction mechanism (i.e., given size and material constraints), a hybrid system can be considered an alternative to increasing the coupling. Otherwise, introducing another transduction mechanism adds loss that counteracts the increased coupling. We show the exact values for a figure of merit, for which coupling is a constituent part, above which a hybrid system is counterproductive.

CONCLUSION

We have analytically determined the maximum possible output power that an energy harvesting system can provide for two cases, when using only one transduction mechanism and a hybrid system with piezoelectric and electromagnetic transducers. We have established the conditions under which one system yields more benefits than the other in terms of power. Especially, a hybrid system is not beneficial and can even cause a decrease in the maximum output power if at least one of the two transducers has an effective figure of merit higher than a threshold value of $\mathcal{M}^* \approx 2.38$. The findings presented in this paper can be used as guidance for designing an effective energy harvesting system.

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